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A More Capable Student in a Meaningful Learning Environment: A Lesson Study Research on How to Develop Students' Mathematical Self-Efficacy

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Abstract **– Self-efficacy affects the students' perceptions of their own competence in mathematics. The learning environment can influence the student's self-efficacy, but also the experiences of meaningful learning. This study examined changes in mathematical self-efficacy in Finnish 6th-grade students through six lesson study processes. Research aimed to answer the questions: "How did students' self-efficacy develop in mathematics learning?", "How did the elements of meaningful learning appear in students' learning experiences? and "What kind of learning environment does support meaningful learning in mathematics?" Qualitive content analysis concentrated on the analysis of changes in self-efficacy in surveys and themed interviews. Most changes happened in the self-efficacy sources of vicarious learning and social persuasion. The changes showed that when a student received cognitive and social support for learning through group work and also continuous feedback from teacher student's mathematical self-efficacy strengthened. This produced meaningful learning experiences which can be considered when designing meaningful learning environment.**

Keywords **– Mathematical Self-Efficacy, Meaningful Learning, Lesson Study, Problem-Based Learning, Learning Environment.**

I. Introduction

Mathematics differs from other subjects due to its abstract and interrelated concepts [1]. This poses certain challenges to the deeper understanding of mathematical concepts. Eventually, this challenge of understanding may make students experience that they do not know mathematics and their faith in their own abilities becomes shaken [2].

When talking about meaningful learning, the word "meaningful" can be understood, first, that the student should feel that learning is personally meaningful. When a student gets the experience that he or she can and does influence his or her own learning, he or she experiences feelings of meaningfulness from his or her own learning [1]. However, meaningful learning can also be viewed as a process of learning that makes sense and feels purposeful. The result of meaningful learning lies in its cognitive residue, the learner' mental model, emphasize Jonassen and Strobel [3].

In this research, we focus on the process of meaningful learning and especially on how to make learning of mathematics, despite its abstract language, meaningful. A favorable learning environment supports commitment to learning and long-term work, maintained by the student's intrinsic motivation. Furthermore, when learning is perceived as meaningful, positive experiences of self-efficacy are created for the student [4].

II. Theoretical Background

A. *Meaningful Learning in Mathematics*

Meaningful learning has been studied widely in the context of mathematics teaching, especially in didactics. For example, educational psychologist William A. Brownell paid particular attention to the importance of mathematical understanding rather than mechanical memorization [1, 5, 6]. In addition, he emphasized the dimension of relevance, which lays the foundation for an experience of meaningful learning [1, 5].

In the field of educational psychology, David P. Ausubel [7] developed Brownell's theory of meaningful learning so that he divided meaningfulness into two dimensions; affections and cognition. Meaningful learning arises primarily from the desire to learn, as well as from the assimilation of new knowledge into the old, already learned skills and knowledge [7]. As a result of meaningful learning, “new psychological and individual meanings” are formed for the student [1, 7, 8]. In order for meaningful learning to arise, the content to be studied should primarily be meaningful and the concepts related to the content should relate the new to what has already been learned [7].

Secondly, the student must be ready to make the effort and actively be present in the learning process, in which new concepts are interpreted and reworked into their own existing conceptual system. Learning becomes a personal process in which, at its best, taking responsibility for one’s own learning and the role of self-leadership increases [7]. Thirdly, Ausubel [7] emphasized the importance of the students’ “cognitive drive”, which creates curiosity about the subject being learned and keeps the student interested, focusing on the learning task itself. This is how studying itself and the task solution can bring pleasure, and the student does not need external motivation [1, 4, 9]. Likewise, Carl R. Rogers [10] pointed out that the criteria for meaningful learning are met if learning is learner-oriented.

The role of the teacher, who is able to form a learning environment conducive to meaningful learning, should also be noticed [10]. The theory of meaningful learning in mathematics by Pyotr Galberin [11] focused on the teaching-learning process and guiding learning. According to Galperin [1], learning should take place through internalization. The student does not learn a new thing just by observing, but illustrative and functional material is also required to internalize a new thing. Galperin’s theory emphasizes the core concept of “orientation”, which is understood not only as a predictive function, but also as an orientation to action [11, 12, 13]. At the orientation stage, the basis of the activity is formed and the subject and goals of the activity are defined, as well as the initial motivation is formed. The material stage is itself a concrete action that is carried out with the help of illustrative means. In the spoken stage, the function itself is described by concepts, speaking aloud and writing. In the phase of internal speech, concrete action moves to the level of personal thinking when the internalizing phase takes thinking to the mental level. Finally, action becomes automated. According to Koskinen [1], the most important principles and working methods are orientation and motivation, illustration, concrete work and use of functional materials, research tasks and problem solving, group work and communication, as well as evaluation that enables the internal learning process.

B. *Mathematical Self-Efficacy*

Students’ have various beliefs of themselves as students and learners. Self-efficacy defines the level of how optimistic the students are about their success and ability learn, and how they approach a task [14]. In the context of school and learning, we can also use the concept of academic self-efficacy beliefs to analyze students’ capability to succeed in academic pursuits [15, 16]. Then the focus is on how confident students are that they will succeed in school. This kind of self-efficacy beliefs have numerous impacts to students’ academic careers and choices they make also later in life [17, 18].

Similarly, mathematical self-efficacy has mainly been studied from two different perspectives. The first examines the effects of mathematical self-efficacy on study and career choices [19, 20, 21]. The studies reveal, for example, the clear impact of mathematical self-efficacy on the choices of mathematical science majors instead of minor subject studies, as well as on success in studies and study attainments.

Another perspective examines the relationship between self-efficacy beliefs and learning outcomes and motivation [22, 23, 24]. Mathematical self-efficacy has been examined especially in the context of problem-solution-oriented learning, self-regulation, and learning strategies [14, 25, 26]. Studies show that mathematical self-efficacy strongly predicts the ability to tackle even challenging tasks, even if the performance does not correspond to the belief in it itself. This is also influenced, for example, by the perceived relevance of tasks in solving individual mathematical problems.

Self-efficacy plays a major role in students' commitment to their learning environment and the activities set there [9, 27]. Students with relatively good self-efficacy are able to engage with learning better in terms of behavior, cognition, and motivation. Higher self-efficacy supports commitment to activities and thus learning and achievement of goals, as well as better learning outcomes [27]. Self-efficacy research has, indeed, mainly focused on the strengthening of self-efficacy and school performance [28, 29, 30], although self-efficacy beliefs play a major role in the regulation of individual learning [14].

The students' self-efficacy is contextually bound and self-efficacy can be developed [14, 27]. Also, the teacher's self-efficacy matters. Namely, according to Tschannen–Mora and Woolfolk Hoy [31], a teacher with good self-efficacy has a positive effect on the development of a student's self-efficacy. The teacher's positive self-efficacy beliefs are particularly evident in three different areas: classroom management, the use of teaching methods, and enhancement of the student's participation in the learning environment [31, 32]. In addition to this, the teachers should show empathy toward their students, as the emotional support provided by the teacher has been found to affect the construction of the students' self-efficacy [33].

III. Method

The purpose of this research was to create a leaning environment that would support students' meaningful learning and their mathematical self-efficacy. The following research questions were set for this study:

(1) How did students' self-efficacy develop in mathematics learning?

(2) How did the elements of meaningful learning appear in students' learning experiences?

(3) What kind of learning environment does support meaningful learning in mathematics?

This was a qualitative research that applied the lesson study (LS) process. It is an interactional development process during which the development takes place in a structured cycle [34]. LS is a collaborative method that has similar features with design-based and action research, and that primarily focuses on the teacher's pedagogical and didactic skills or on the teacher-student interaction and student's individual observation. Thus, LS is a research method that aims to develop teachers' and students' abilities.

In this research, LS was implemented as six cyclic processes aiming to provide students with various self-efficacy-related experiences. The first cycle was a LS pilot. All cycles followed the phases of goal setting, lesso-

-n planning, research lesson, post lesson discussion and reflection [35].

The research was conducted during mathematics lessons among sixth-graders (N = 18, aged 11-12 years) in a northern-Finnish school during 2018-2019. Lessons were taught with the co-teaching method where the elementary school teacher and mathematics subject teacher taught students two hours once a week. The elementary school teacher was also the teacher-researcher of this study.

The lesson structure followed mainly the Japanese problem-based lesson model [36], presented in Table 1.

Table 1. Problem-based lesson model in research lessons.

Problem-Based Lesson
1. Mathematical dialogue between teachers – Teachers presents the topic through dialogue by using the actual concepts.
2. Differentiated individual problem-solving- Pupils can choose tasks from three different levels and solve them individually. Teachers help and diagnostically evaluate pupils learning.
3. Sharing the individually solved tasks in groups.
4. Authentic problem-based task in groups.
5. Sharing the solved problem between the groups.
6. Choosing the best problem-solving strategy - Pupils discuss and justify the most interesting solution strategy for authentic problem.
7. Reflective discussion between teachers.

Research permissions were obtained from the students' guardians. Ethical questions were discussed during the whole research process carefully because the participating children were underaged and entering their puberty. Anonymity was paid special attention by coding children in the data with random pseudonyms and anonymizing the data [37].

Each LS process focused on different mathematical substance area (as determined in the Finnish National Core Curriculum for Basic Education) and had special work methods, but otherwise each process followed the same lesson structure. The co-teachers reflected their observations after each lesson together and did necessary revisions based on their experiences for the next lessons. Thus, the activities developed all the time during the LS process.

The data collection process is described in Table 2. Students' self-efficacy was monitored with surveys that were conducted six times. The number of survey responses varies because a student could be absent when the survey took place. In addition, two themed-interviews related to the elements of self-efficacy were conducted.

Table 2. Data collection process.

Autumn 2018	**Substance**	**Working**	**Data collection**	**Amount of data**
Lesson Study 1 (pilot) - 6 research lessons	Basic arithmetic	Homogenic group of six	Self-Efficacy survey - initial phase Reflective discussions and researcher´s notes	17 survey answers 6 reflective discussions
Lesson Study 2 - 6 research lessons	Decimals	Heterogenic group of three	Self-Efficacy survey Reflective discussions and researcher´s notes	14 survey answers 6 reflective discussions
Lesson Study 3	Geometry	Favorite pair	Self-Efficacy survey	15 survey answers

Autumn 2018	Substance	Working	Data collection	Amount of data
- 6 research lessons			Interviews Reflective discussions and researcher´s notes	17 interview answers 6 reflective discussions
Spring 2019				
Lesson Study 4 - 6 research lessons	Measure	Independent	Self-Efficacy survey Reflective discussions and researcher´s notes	16 survey answers 6 reflective discussions
Lesson Study 5 - 6 research lessons	Fractions	Heterogenic group of six	Self-Efficacy survey Reflective discussions and researcher´s notes	18 survey answers 6 reflective discussions
Lesson Study 6 - 6 research lessons	Acreage and amplitude	Heterogenic group of six	Self-Efficacy survey – final phase Interviews Reflective discussions and researcher´s notes	18 survey answers 18 interview answers 6 reflective discussions

The self-efficacy survey was an application of self-efficacy scales [14, 38, 39]. Three questions were constructed by each self-efficacy sources; personal performance accomplishments, vicarious learning, social persuasion, and physiological states and reactions. The survey used the Likert scale of five with very uncertain, slightly uncertain, I cannot say, slightly sure and very sure.

In addition, the researcher interviewed the students twice; first time in late autumn 2018 and second time in late spring 2019. The interview was a thematic interview that followed the theoretical framework of self-efficacy. A total of 12 questions were asked in both interviews. Furthermore, in accordance with the Lesson Study method, after each research lesson, the research teacher and the mathematics subject teacher held a reflective feedback discussion on the observations of the students' actions, on the basis of which the research teacher recorded notes.

The analyzing method was a theory-led, qualitative content analysis method [40]. The self-efficacy theory guided the analysis strongly, as the survey and interviews were designed based on the theory. Within the theory-led contents, the analysis got inductive elements as the findings emerging from the data were highlighted in the analysis. The analysis was divided into three phases: first two phases focused on the research questions 1 and 2, while the third part of analysis resembled a synthesis answering to the research question number 3. The first part of analysis focused on the changes in self-efficacy. A comparison table of all 18 student's self-efficacy survey responses was built. The comparison showed differences in changes of successes and mastery experiences, model learning, feedback and support from others, and feelings and experiences and their interpretations. Then, the analysis continued by coding self-efficacy changes one student at a time. At this phase, the interview data was accompanied in the analysis. Next, the data was categorized which resulted in ten themes.

The biggest changes were found especially in the sources of vicarious learning and social persuasion. The coding of the material continued so that the students' changes in self-efficacy began to be coded one by one. Student-specific change tables recorded excerpts from transcribed interview material, which was used to examine changes in self-efficacy in more depth.

The research question number 2 was answered by coding from the data all references to meaningful learning. The analysis showed that two categories could be found that formed the main findings related to students' perce-

-ptions of meaningful learning. In the third phase, the analyses from phase 1 and 2 were combined and viewed from the perspective of a meaningful learning environment.

IV. Findings

The results are described by combining the features of the changes that have occurred in the different sources of self-efficacy and the meaningful learning that occurs in them. The results were found from the student´s evaluations and experiences of learning in mathematics, which were reflected from the data. First, changes in students' self-efficacy are examined in four different areas of self-efficacy, after which features of meaningful learning in students' experiences are highlighted.

A. *Significant Changes in Students' Mathematical Self-Efficacy*

a. *Personal Performance Accomplishments*

Based on the surveys, there were hardly any changes in this area of self-efficacy. However, 17 out of 18 students described their own experiences of learning and success in a positive tone. In the thematic interview, all students were asked what experiences of success they have experienced during the past school year and what they have learned. In the interviews, some students described their own experiences clearly related to changes in substance competence. The experiences were clearly partly related to a single mathematical area.

I guess the best feeling was when I realized how to calculate the surface areas. (Ernest)

I may have learned some divisional calculations. They were difficult for me last year because I didn't understand what they kind of were. (Allison)

On a general level, students experienced the highest level of self-efficacy in learning fractions and acreage and amplitudes. The reason that they felt most capable of learning these very areas may be that they were the last to study. As a result, many students had probably developed a stronger belief in their own learning possibilities. On the other hand, it may also be the case that the students identified concepts related to these areas in the surveys. This may explain the fact that they also, in principle, already understand a little of the substance being studied.

The students described the insights related to substance competence on a general level, which were clearly not tied to just one area of competence.

When I got a problem-solving task done. (Victor)

When I learned something and I got a new calculation done. I'm developing at calculating speed. (Wera)

Experiences of success related to substance competence can also be accompanied by clear task- or textbook-oriented experiences. Success is accompanied by a clear goal, which means that completing something to the end produces experiences of pleasure. The focus is then largely on the result or technical execution.

When I had everything right in extra tasks and homework. (Jack)

It´s been nice if I managed to do all the tasks right. (Isla)

Another theme that emerged was experiences related to success. When the student focuses on the result, grade or reward motivates him to learn.

I had 10- or 9,5 from the test, that made me really happy. (Henrik)

The test is perceived as a measure of competence and, for example, the fact that the student performs in the test situation or has been able to complete all the tasks in the test reflects to the student´s perception of good competence.

Well, when I feel that the exam went well and I'm happy with it. (Minnie)

If an exam was difficult but I could still complete it. I could answer all questions. (Peter)

The experiences of the studying showed that some of the students had realized the importance of working hard. There were also indications of the importance of the learning process itself and the depth of the matter. A certain kind of peer pressure can have a positive effect on the student's delving into the matter, especially if the other students in the group show persistence.

My persistence has developed. (Thomas)

Before I thought a lot that I can´t do Math, but now if I succeed in something it gives me motivation. (Sally)

Now I know more how to process because before all I wanted to achieve was the solution as fast as possible. I have realized that I don´t need to learn everything right away. (Sally)

Yeah, once we were processing one difficult task with the group quite long time. And then, when we got the task right it made me feel very good. (Don)

The development of working skills is seen as an important part of the learning process and the breaking of one's own preconceptions. The student has gained experience related to different working skills and is able to evaluate a suitable way of studying. This is important, because the student does not have to focus on maintaining working skills or experience social pressure to perform, but can concentrate and delve into the study of substance. Understanding one's own way of learning is also strongly associated with a sense of control, which is also reflected in the student's confidence in their own ability to learn mathematics.

I have developed in group working skills. Now I manage participate also with boys and only with girls. (Tim)

Independent working suits for me. I have my own space and peace to work. (Jack) In a way, independently, you may be able to reflect on that answer in peace. If there is a group where there are many people who know how to do it, then I feel like I have to get that answer very quickly. (Sally)

b. *Vicarious Learning*

In terms of vicarious learning, the biggest changes occurred with the item "I learn better in a group than alone" and "my role in group assignments is important". In the initial survey, 9 of the students and in the final surveys 13 answered with some certainty or absolute certainty that they learned better in a group than alone. In interviews, 15 out of 18 students responded that they think the best way to learn is in a group. In interviews, 15 out of 18 students responded that they think the best way to learn is in a group. This may well suggest that the surveys were influenced by close-knit experiences that students mirrored in their responses. In the interviews, it was easier for the students to give an overall picture of their experiences.

Another vicarious learning change was related to the statement "my role in group assignments is important". The responses are described in Table 3.

Table 3. "My role in group assignments is important".

Proposion: My role in group assignments is important		
	Initial survey	**Final survey**
Very uncertain	0	2
Slightly uncertain	0	0
I cannot say	9	5
Slightly sure	7	9
Very sure	1	1

The students felt that the role of group assignments was more important in the final survey. They were able to give their opinion on the matter more boldly, because the "I cannot say" responses halved from the initial survey. In the interviews, the students described their role in many ways, such as feeling involved in reflecting on the tasks and an important part of the solution achieved by the group together, or feeling that working in a group was meaningful.

In the interview questions related to vicarious learning, the researcher wanted to find out in what way and why the students felt learning mathematics best. The significance of interaction in the student's learning process was strongly highlighted in the material. The vast majority of responses showed that, as a rule, students felt that they learned best in a group. This also supported a strong change in self-efficacy surveys in how students felt they could learn best in a group than alone.

The experiences of students were clearly divided into two different themes, which show the importance of interaction and vicarious learning in learning and studying. The first theme is social support, which emphasizes the opportunities designated by the student to discuss and reflect on mathematics together, as well as getting help and support. Understanding different perspectives and strengths can take learning forward and student's understanding of what is being learned expands.

Working in group is nice because you can reflect together with friends. (Ernest)

In the group working is nice because there are many people who to talk to and compare the solutions together. (Kaya)

If you don´t know the solution, you can try to figure it out together. (Sally)

However, it is important that the answers are genuinely considered together and that cognitive exchange takes place. If interactive learning is too much about doing for the other person or giving direct answers without looking at all points of view, it can, at worst, hinder learning.

Working with pair is nice if that pair is not doing too much things for you but will help if I don't understand. (Sally)

Social support was considered important precisely in terms of getting help and emotional support. Thus, when a student has the opportunity to rely on a peer or members of his or her group, he or she does not feel left alone with the subject being studied and the pressure to perform decreases.

When working in pairs and groups, you don't take so much pressure on whether you can do all the tasks or n-

-ot. (Sam)

Everybody gets help in the group. (Ada)

I can concentrate better in the group because I know I will get help. (Isla)

Allison pointed out that working in a group means not only dealing with topics related to substance and solving problems, but also how to act and behave in a group. She developed the experience that leading a group is of great importance so that the activity proceeds well. She values it as even more important than being able to teach or help others.

I don't know how to help others but I know how to lead my group. (Allison)

Cognitive support was another theme that emerged from experiences related to the model learning area. The student feels that they receive support for learning when there is an opportunity to compare different techniques and perspectives. You can be sure of your own solutions when more people in the group agree on the same solution.

I learn when we all have different perspectives and solutions. (Allison)

I have learnt different techniques to do the tasks. (Elizabeth)

In the group is possible to hear many perspective and opinion and to be more comfortable about the solution. If there is only like your pair, it´s more uncertain to know if the solution is right…kind of. (Sally)

c. *Social Persuasion*

In the source of social persuasion, the biggest changes came from the item "positive feedback encourages me in my studies". The results are described in Table 4. There was significantly less dispersion in the results of the final survey, and the majority of the students felt that positive feedback mattered.

Table 4. "Positive feedback encourages me in my studies".

Proposion: Positive feedback encourages me in my studies		
	Initial survey	**Final survey**
Very uncertain	0	0
Slightly uncertain	2	0
I cannot say	5	5
Slightly sure	6	12
Very sure	4	0

Another clear change was in the item "math grades matter to my parents" is described in Table 5.

Table 5. "Math grades matter to my parents".

Proposion: Math grades matter to my parents		
	Initial survey	**Final survey**
Very uncertain	0	0
Slightly uncertain	1	1

Proposion: Math grades matter to my parents		
	Initial survey	**Final survey**
I cannot say	5	10
Slightly sure	6	3
Very sure	5	4

The results could probably suggest that the result-oriented nature of the mathematics subject is no longer so strongly guided the students' motivation to study. Consequently, they felt that the value requirements did not necessarily have such great significance for the parents. Secondly, it may also indicate that the grades of a few students increased during the school year, so there has been no need to have a related discussion at home, or the grades simply have not been given that much value in general.

In the interviews, the majority of the students did not point out that the subject had been discussed at home or that the grades would have much meaning. Only one student cited an increase in conversation at home during the school year.

With regard to the social persuasion, the aim was to find out how positive feedback was connected with self-efficacy and what kind of feedback and support were mentioned by the students being important to their studies or learning. The theme of motivation, which has been generated by feedback and support, emerged from the data. The student has felt that the feedback they have received has come from the right reasons or related to some important learning experience.

There has been more motivation to do and I just want to learn more. (Tim)

The teacher has given feedback when I have figured something out. (Sam)

In addition to motivating, positive feedback can affect the student’s self-confidence and increase perseverance. Even if the feedback may not always be positive, it can lead to positive results according to students’ perceptions.

Feedback has made such a difference that it makes you feel more confident. (Kaya)

I've been able to try harder through positive feedback. (Millie)

If I get positive feedback then it motivates me more to study, but also if I get negative feedback I want to show that I can do it. (Sally)

The second theme was the increase in understanding, where the student becomes more aware of their competence and begins understand themselves as a learner.

I have received encouragement from parents and another student. It's nice in that way that you then know how I've been succeeded. (Don)

At home, together with my parents, we have started to think about my homework, for example. It’s nice to hear that what that adult actually knows about it. I’ve learned so much that way. (Sally)

d. *Physiological States and Reactions*

The students reported no major changes in the area of physiological states and reactions. There were fewer st-

-udents who found mathematics a little distressing in the final survey. Most could not say how they felt about it. This was not clear in the interview data either, but it turned out that the students had certainly experienced a huge effort in studying mathematics. At this point, it is difficult to interpret whether it is the students' actual experience of anxiety towards the subject itself or all the work and effort involved in it in general. Based on the students' responses, this area was divided into two different themes, in which the students gave affective or cognitive meanings to the learning of mathematics.

In an affective sense, mathematics seemed like a pleasant subject that did not cause major negative emotions or feelings. The interventions that took place during the school year had helped some students to change their motivation in a more positive direction.

Things have gotten better. More nice and pleasant things have come up. (Ernest) My attitude has changed for the better during this year. (Henrik)

The feelings were not only related to the learning itself, but also the working produced pleasant feelings.

Working in mathematics is fun. (Ada)

When a student gives cognitive meaning to learning mathematics, it can be said that the student is cognitively attached to learning. Sometimes the attachment to learning itself is so strong that the meaningfulness of the subject does not matter. The learning itself produces feelings of pleasure. The student has internalized that math has a meaning in everyday life, which is why it is important to learn the subject.

Well, I don't really like Math, there are some nicer subjects, but Math is really important to know, but it's still not fun. (Kaya)

Well I don't know, at least you learn math for adulthood, for example, when you go to work. This isn't my favorite subject, but it's nice sometimes. (Tim)

The first research question was used to examine what kind of experiences the students had given to learning and studying mathematics when analyzed through the four areas of self-efficacy. The experiences describe their change in mathematical self-efficacy. Next, another research question is answered, which describes the concepts that clearly refer to the features of meaningful learning from the description of the students' experiences.

B. *Features of Meaningful Learning in Students' Learning Experiences*

In the experiences described by the students and the concepts that appear in them, two different trait categories of meaningful learning emerged clearly in the areas of self-efficacy. Categories are covered here through related concepts.

a. *Cognitive Features*

The concepts referring to the cognitive feature category found in the students' comments were features of meaningful learning that the students had experienced in the learning itself and in the construction of knowledge. Allison, for example, described that she had not previously understood a single matter of substance, but had nevertheless gained the experience of understanding this particular issue now. Ernest, on the other hand, described the experience of internalize, which may have been a very significant event in the student's learning process. In an interview, he mentioned this experience when asked about the most memorable positive learning experience.

Several of the students' responses mentioned the importance of understanding the substance, which they described as how they had received the assignments correctly or understood a new thing that has helped to build new knowledge. This is also intrinsically linked to, for example, Ernest's and Sally's experiences of teamwork, where there has been an opportunity to discuss, reflect and compare different perspectives. The experiences of several student highlighted the development of their own working skills, especially in group work. The students had felt that the best way to learn, was together with peers, in interaction.

The students also had the opportunity to learn other skills in interaction, such as leading themselves and others. As Allison described, she knew she couldn't teach others, but she could lead her group. Peer learning, discussion and continuous feedback on one's own competence in a group also help students assess their own competence. It is the teacher's responsibility to shape the learning environment in such a way that it creates opportunities for discussion. The level of discussion should be constantly developed so that students learn to use concepts that are appropriate to the topic. This kind of cognitive collaboration increases the student's consciousness in a versatile way and assimilates existing and new information.

b. *Affective Features*

The concepts referring to affective features emphasized the emotional experiences associated with learning. For example, Millie and Sally described the feeling of trying more than before in mathematics and Thomas spoke directly about the fact that if something has developed in him is perseverance. Several comments also suggested that the students' curiosity and thirst for knowledge have clearly increased.

In his experiences, Don described how, for him, the fact that although the difficult task and solving it took time among the group, solving the task brought pleasure. For example, Kaya mentioned that mathematics itself didn't feel like she liked it but it was still nice to learn and she felt that the subject was important. When the student has understood that as a result of persistent and unyielding work, learning itself can be the best source of pleasure, one can talk about the emotional state of intrinsic motivation. The sense of control produced from competence develops the student's self-confidence, but the student must become aware of their own level of competence. External feedback plays an important role in ensuring one's own emotional experience. Receiving support is also of great importance, because when a student has the opportunity to receive social and cognitive support it is possible to experience a sense of security. Ernest and Ada described how learning and working had been enjoyable and there had been more nice things about studying. This has probably also helped positively with their attitude towards the subject.

Tim was already able to reflect on his studies from the future-oriented point of view that although the subject does not always feel comfortable, mathematics is an important skill in life for the future. In this way, in addition to the desire to learn, a new kind of dimension is transferred to one's own studies, which helps the student to set goals for himself and for his learning.

V. Discussion

Based on the findings of this research, a meaningful learning environment [1, 10] can be created in order to develop students' mathematical self-efficacy. Furthermore, students' self-efficacy can increase students' engag-

-ement to their own learning environment and tasks [27].

In this research positive changes took place in students' self-efficacy as described by the students themselves. Their experiences also included features of meaningful learning. According to Linnenbrik and Pinrich [27], students' self-efficacy is always context-bound and it is possible to develop it within the context learning takes place. However, it should be noted that self-efficacy and reported positive changes are not necessarily permanent but positive experiences and awareness of the development of one's skills have a significant effect [14]. Furthermore, negative experiences and lack of emotional regulation especially when failing may lead to a situation where a student perceives mathematical skills as his or her permanent feature and starts to believe that he or she cannot learn mathematics [41].

In this research, the changes in students' self-efficacy that the students' brought up happened specially in the areas of model learning and feedback and support from others. According to Bandura [14], a core element of developing self-efficacy is social comparisons especially when students are not yet aware of their skills and abilities. Students' awareness is built by observing class mates and comparing others' performing with their own. In data, the students described that immediate feedback and support from the teacher helps them to try harder and it creates confidence. Mathematical self-efficacy is supported and developed by realistic and truthful feedback that comes from a reliable sources [42]. The student must be made aware of what entrepreneurship accomplishes [42]. However, in order to make this possible, it is necessary to have sufficient adult resources in mathematics. For this reason, continuous and individual teacher feedback and support given in the research classes was possible, as there were always two teachers in all research classes. One teacher does not have time to guide students sufficiently individually, especially when the levels of learning, working, and interaction skills between students are so different. Adequate support creates a sense of security for students.

According to this study, a meaningful learning environment in the subject of mathematics is one in which features of meaningfulness at the cognitive and affective levels. The cognitive level is supported, in particular, by the importance of developing the student's mathematical thinking and understanding. According to Galberin [11], learning should take place through internalization through the stages of orientation, material, spoken, inner speech and internalization. Understanding and internalization can only occur through interaction and speech communication with others [11]. So at the cognitive level, it is necessary to take into account the ways of working that allows interaction. Students should be given opportunities to discuss, compare and reflect on different perspectives.

Simply enabling interaction does not yet produce meaningful learning in the subject of mathematics. Due to the abstract concepts and substance typical of mathematics, it is important for the teacher to be aware of the importance of understanding and internalization [1]. According to Ausubel [7], meaningful learning experiences can arise if the substance being studied feels meaningful and the concepts are sufficiently comprehensive but understandable. Without these qualities, cognitive assimilation becomes difficult and abstract, and without a deeper level of understanding, the student may develop the experience of weak self-efficacy. In the research lessons, in addition to enabling interaction, understanding was supported through tasks of different levels, in which the concepts to be learned were constantly visually visible. Problem-solving tasks were always attached to some practical context that helped students perceive the practical meaning of mathematics. A wide range of concrete tools were used to support learning and perception, on which the students had the opportunity to rely.

On an affective level, a meaningful learning environment must support the student's perseverance, where the desire for knowledge and learning new things motivates the students to study. According to Koskinen [1], the student must be prepared to make an effort and actively be present in the learning process, in which new concepts are interpreted and reworked into their own existing conceptual system. Not only learning, but also the development of perseverance requires the necessary amount of repetition. A recurring structure was created for the research lessons in each class, which made it possible to work independently and in a group. The students had the opportunity to focus on studying mathematics and did not spend too much time adapting to the environment. This was manifested precisely in the task-specific experiences of meaningfulness. The only stage of adaptation came at a time when the size or composition of the groups were changed.

When a student is sufficiently attached to learning and studying, learning itself can produce pleasure and experiences of meaningful learning, even if the subject is not so interesting. The desire for knowledge may be detached from the meaningfulness of the subject, in which the student purely experiences curiosity about, for example, the solution of the learning task. On the other hand, as emerged from the data, learning can be strongly motivated by understanding the relevance of the subject. Therefore, the things that matter are the most meaningful.

Limitations of the Study

This study focused on looking at the change that occurs within a single context, which does not provide a fully comprehensive research result. Also, when studying students, the researcher must consider changing circumstances and changing emotional states. Each lesson is influenced by a variety of factors that, in turn, pose a challenge to the researcher in relation to the reliability of the research [43]. The results of the students' self-efficacy surveys must consider the students' current and pre-existing situations and feelings. In many places, the students' mood may have influenced their own beliefs in self-efficacy in general, and this has also affected self-assessment in mathematics. The experiences described by the students in the interview data are also completely linked to their current emotional state, school day or life situation. For example, before the first interview, a student was in such a bad mood that he refused to answer the researcher's questions. These should always be considered when examining children and adolescents.

Efforts were made to ensure the reliability of the study through methodical, researcher, and theory triangulation [44]. The study used two different data collection methods, surveys and interviews. Although the survey did not enable any statistical analyses, the changes that occurred in the students' answers could be further examined in the interviews. The researcher teacher who was mainly responsible for the research was closely involved in the study, but also her concurrent pair of teachers, who constantly observed and reflected on the activities.

VI. Conclusion

This research was conducted in the form of co-teaching that helped the teachers develop their professional skills within the context of their own work. It also gave them courage to test the pedagogically new method of teaching mathematics. Colloquial exchange and goal-setting appeared an advantage because the subject teacher knew well the mathematical substance while the elementary school teacher had good knowledge of the students and classroom management skills. Together they formed a pair of teachers with relatively good self-efficacy, which was considered important for the objectives of increasing students' participation and motivation and creating an encouraging learning environment [45, 46]. Moreover, according to Chong and Kong [47], teachers' high self-efficacy can create a learning atmosphere in mathematics that spurs students to seize even challenging tasks and to believe they will succeed.

Working as co-teachers also seemed to provide more resources to observe students' learning, give formative feedback, and support students individually. When it comes to further needs for research, this study inspired the teachers to develop their own expertise in relation to the that of students within an interactional learning environment.

This research had a strong focus on developing self-efficacy in a school context. For further research, it would be interesting to more strongly link the importance of the home in changes in the student's self-efficacy beliefs. In this way, a more holistic model could be developed to support the student's mathematical self-efficacy and meaningful learning.

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