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Secondary School Students' Conceptions of the Roles of Proof in Trinidad and Tobago

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***Abstract* – This study examines students' conceptions of the role of proof in Trinidad and Tobago. I conduct semi-structured interviews with 21 secondary school students, ages 13–16 years old, to investigate their opinions of (a) the purposes of proof in mathematics and (b) the type opportunities for proving in geometry in the Caribbean Secondary Education Certificate (CSEC) examinations. My analysis suggests that students identified the roles of verification, explanation, systemization, and appreciation within the work of mathematicians or school mathematics. The latter role suggests students' understanding of the intellectual need of proof in their mathematical learning. All 21 students considered the calculations with explanations questions in the CSEC examinations as informal opportunities to construct proofs. The development of these non-proof arguments has the potential to go beyond the borders of this reasoning and proof activity to evolve into proof construction. These findings can provide researchers with possible evidence of students' learning with regard to the recent reform-oriented mathematics curriculum in Trinidad and Tobago.**

***Keywords* – Curriculum, Reasoning and Proof, Secondary School Mathematics, Students' Conceptions.**

I. Introduction

The reformers of mathematics education in Trinidad and Tobago suggest that students should have more opportunities with proof in their secondary school mathematical experiences (Republic of Trinidad and Tobago, 2009). In Trinidad and Tobago, researchers are working to understand students' perspectives about what constitutes a proof and the roles of proof in school mathematics. Researchers are also interested in how the new reform-oriented mathematics curriculum, Caribbean Secondary Education Certificate (CSEC) examination materials, and teachers' instruction influence students' notions of proof in school mathematics. To date, there are no existing studies, which examine students' mathematical learning in the post-implementation period of the reform-based curriculum. Furthermore, the recent CSEC examinations offer more calculate and explain type of questions rather than directly asking students to prove (CXC, 2014). As a result of these concerns, there exists a need to investigate students' perceptions of (a) the purpose of proof in school mathematics and (b) their opinions of the opportunities to do proofs in their textbooks and CSEC assessment materials. Based on the need to investigate students' perceptions of proof and its role in their mathematical learning experiences, the following research questions drive my inquiry:

RQ 1: How do secondary school students in Trinidad and Tobago view the roles of proof in mathematics and school mathematics.

RQ2: What are students' conceptions about the type of opportunities for proving in geometry in the CSEC examinations?

II. Theoretical Perspectives

In considering students' perspectives on Geometry proofs, past studies demonstrate that students either do not understand the role of proof in their mathematical experiences or do not appreciate the necessity of proof in school mathematics (e.g., Chazan, 1993; Herbst & Brach, 2006; Kunimune, Fujita, & Jones, 2009; Schoenfeld, 1985). For example, Kunimune, Fujita and Jones (2009) reported that although most 14 – 15 - year - old students in Japan (in the third year of secondary school) could write a proof, around 70% could not understand why proofs were needed. The students felt that the purpose of writing proofs was for demonstrating one's knowledge of previously taught theorems. This finding is in agreement with other studies, which also found that Geometry students view opportunities for reasoning and proof as arbitrary exercises (e.g., Chazan, 1993; Fischbein, 1982; Herbst & Brach, 2006; Schoenfeld, 1985; Tinto, 1988). In Herbst and Brach's (2006) study with 16 high school students enrolled in two accelerated Geometry classes, the students claimed that the purpose of reasoning and proving was to provide opportunities for them to showcase their reasoning and communication skills. In Schoenfeld's (1985), students also viewed Geometry proofs as simply arbitrary exercises in logic that merely confirm results already known to be true. Additionally, in Tinto (1988) students viewed reasoning and proving activities as exercises imposed on them by their teacher or textbook. Finally, McCrone and Martin (2009) found that students in their study also considered the purpose of proof to be applying recently learned theorems and not as a mathematical process for establishing the truth of theorems. Although the aforementioned studies were conducted with different populations of students from different educational systems and background, these findings provide evidence that students may not see the intellectual need for deductive arguments in their mathematical experiences.

Other studies also indicate that students may not also fully understand the role of deductive arguments in proving (e.g., Chazan, 1993; Fischbein, 1982; Healy & Hoyles, 2000). These studies show that students still considered the possibility of finding a counterexample after seeing a correct deductive argument. This indicates that students fail to see the intellectual link between deduction and justifying general claims. Students may not understand the pivotal role a valid deductive argument plays in assuring the claim holds for all cases.

The aforementioned findings indicate that students may not perceive the intellectual role of reasoning and proof in mathematics. These findings highlight the potential danger of students not appreciating the necessity of reasoning and proof in their mathematical experiences. Research has shown that such perspectives can be unfavorable to students' future learning of mathematics (e.g., Muis, 2004). Therefore, it is important that in the case of Trinidad and Tobago, that we investigate Geometry students' views of reasoning and proof in school mathematics. For example, students may consider reasoning and proof as an arbitrary activity imposed on them by their teacher, examiners, and textbooks. This could be a possible explanation for their low performance on proof-based items in CSEC examinations. Furthermore, the findings of my study can potentially help us determine whether students in Trinidad and Tobago see the intellectual need for reasoning and proof in their mathematical experiences.

In this study I use a framework previously used by McCrone and Martin (2009) and Dreyfus and Hadas (1987) to help investigate students' conceptions of proof. This framework entitled: *The Six Principles of Proof and Understanding* describes the knowledge any person within an informed mathematics community should possesses about the roles, structure, validity and generality of a proof. In this brief report, I discuss the findings associated with the roles of proof in mathematics, school mathematics, and assessment materials.

III. Methodology

In this study, I conducted semi-structured interviews with 21 students (male or female who are 13 to 16 years old) in forms three (ages 13-14), four (ages 14-15), and five (ages 15-16) from three selected school sites. For each school, I randomly selected seven students from those who provided voluntary assent and parental consent to participate in the student interviews. Additionally, these students also participated in the proof-based lessons on Congruency of Triangles, I observed in another study examining the teaching of reasoning and proof, (Hunte, 2016). The selection of the seven students from each school was according to the grade level I observed at the respective school. Each interview lasted 45 minutes to one hour. Typical questions I asked interviewee's, were: "In your opinion, why do you think mathematicians write proofs?" and "In your opinion, why do you think you are taught or shown proofs in your mathematics classes?" Students in their responses articulated their opinions according to the questions. I transcribed and analyzed each recorded interview and coded the responses according to recurring themes aligned with the roles of proof evident in students' responses. A fellow researcher facilitated reliability coding of five randomly selected interview transcripts. We obtained a 96% reliability of themes within students talk about the roles of proof in mathematics, school mathematics, and assessment materials.

IV. Results

According to students' responses, the roles of proof in mathematics included verification, explanation, and systemization. In Table I, I present a summary of the meaning of each of these roles of proof students identified in mathematics. I also present counts of the number of students identifying each role. These counts do not indicate the number of times a student mentioned a role but the number of students who talked about a specific role.

Table I. Roles of Proof in Mathematics.

Role of Proof	Description	No. of Students
Verification	To verify that a statement or a conjecture is true	9
Explanation	To give insight into why a statement is true	15
Systemization	To build an axiomatic system of results	2

A. *Representative Quotes of Student's Conceptions of the Roles of Proof in Mathematics*

When I asked the question "In your opinion, why do mathematicians write proofs?" The following students, stated:

Ava: "*Mathematicians write proofs to explain. I mean that proof explains the things a lot easier for other people like other mathematicians and people who will read the proof see exactly how other mathematical concepts make up reasons supporting why the result is true rather than just showing it is true why the mathematical concepts make up reasons supporting why the result is true rather than just showing it is true.*" (Explanation).

Ray: "*To back up themselves to show they are right and really this is important because of how mathematics topics are usually connected, he may need to verify that his Theorem is right so that he can use it to prove other results later on.* (Verification & Systemization).

B. *Roles of Proof in School Mathematics.*

In their responses, students identified two roles of proof in school mathematics. By school mathematics, I refer to the proof opportunities in school during the teaching of various secondary school mathematics topics. Students stated the following roles (a) promoting understanding and (b) appreciation of mathematics. Of the 21 students, 19 students talked about the promotion of understanding. According to the students, three main themes emerged in their discussions of this of this role of proof. Students claimed that through the promotion of understanding, they can develop (a) insight into why a theorem is true, (b) knowledge of the utility of a proven result, and (c) habitual inclinations for their own proof writing practices.

Of the 21 students I interviewed, 5 students talked about the role of appreciation. This role is an interesting finding because it demonstrates a unique conception of proof held by students in Trinidad and Tobago. According to these students, the teaching of proof allows them to appreciate the usefulness of the underlying axioms and other mathematical results used to construct a proof. For example, when I asked, "In your opinion, could you provide reasons why you are taught proof in mathematics?" the following students explained:

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Danni*: "I think Miss really shows us the proof even though it is not required, to help us appreciate the mathematics you are learning like where it came from and how to use it to solve problems."*

Melissa: "*Well the teaching of proof helps us to really value the usefulness of mathematics when you see how Miss [teacher] apply some other results we learned before to prove some result we are now learning."*

Taylor: "*To help us understand and appreciate the mathematics we are doing cause if she does not show us the proof for a result we will just write it down and not really understand the purpose of it and how to connect it to other stuff and we would really be excited about it.*"

Overall, these students demonstrated through their opinions that the appreciation of mathematics is an important component to their learning in school. Each student articulated how the role of appreciation motivates an awareness of the usefulness of their pre-existing knowledge and the application of a new result. In particular, Taylor emphasized that their appreciation of a new result allows her to see the connections of new content to previous knowledge. Melissa emphasized that through her teacher's application of previous knowledge when writing a proof, she valued the usefulness of her previous knowledge to verify and explain the existence of a new result. As a result, Melissa noticed the links between her previous knowledge with the new knowledge she learned. Danni asserted that through the appreciation of a mathematical formula, he became aware of its' utility during problem solving.

C. *Opportunities for Proof in External Examinations*

In the introduction of this paper, I highlighted the prevalence of calculate and explain type of questions in the CSEC (external examination body in Trinidad and Tobago) mathematics examination. In recent years, there has been the prevalence of reasoning and proof items in the CSEC mathematics examination Geometry questions (CXC, Subject Award, 2013, 2014). Figure 1 shows a typical exam question in the January, 2014 examination. In this question, students are asked about finding the unknown angles *x, y,* and *z* in the given diagram. Students are expected to use the given information that the lines *AQB* and *CRPD* are parallel where *MQRN* is a transversal (cutting across the aforementioned parallel lines) to help solve for the unknown angles. Additionally, students are provided with the measure of angle *QPR* and the fact that the lines *PQ* and *PR* are of equal length. This question required that students calculate the unknown measures and provide reasons justifying their computations.

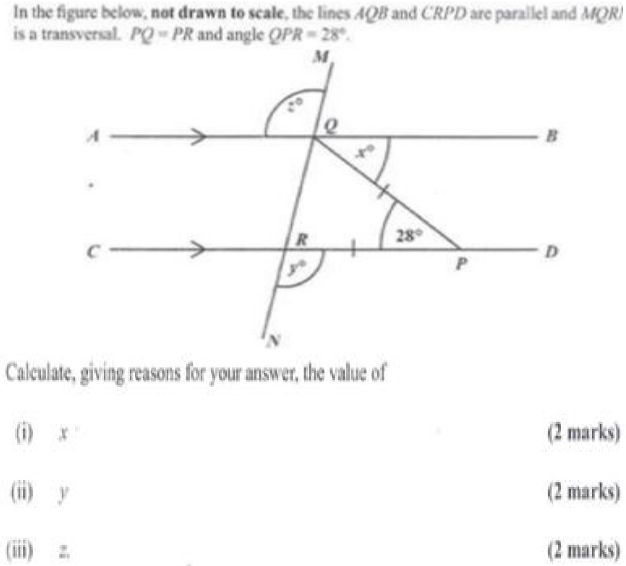

Fig. 1. An example of reasoning and proof question from CSEC January 2014 mathematics examination.

According to the examiners' report, this question assessed students' ability to determine the measures of angles using the properties of parallel lines and transversals. Of the 94% of students attempting this question, less than 1% earned the maximum marks. The examiners reported that students' performance generally was unsatisfactory. Overall, students were able to correctly state the values for the unknown angles in part (i) angle *x* and part (ii) angle *y.* However, over 90% of these students were not able to state the value of angle *z* in part (iii). In the cases where students were able to identify the measures of all the unknown angles correctly, they could not provide reasons to support their calculations. The examiners recommended, "candidates [students] should be drilled in the practice of stating reason or reasons for answers derived from the Geometry of plane figures" (CXC, 2014, p. 8,). The aforementioned quote suggests that teachers should provide opportunities for students to develop supporting arguments, which use their Geometry knowledge to justify the validity of calculations they perform. The examiners also recommended that teachers should encourage their students to use mathemati-cal terms to describe the geometrical relationships they observe or derive from geometrical figures.

Although this question did not explicitly ask students to prove, students were expected to provide supporting arguments using the properties of lines and transversals to justify their calculation. A typical response to this question can be considered as an opportunity for reasoning and proof, I define as a Geometric Calculation with Number and Explanation (GCNE) (Hunte, 2018). A GCNE is a geometric calculation that requires the use of geometrical theorems and definitions to explicitly provide reasons to support the result of the computation. Therefore, it is of interests to this study that with the prevalence of such opportunities in the CSEC exams, I investigated whether students realized that these informal arguments could be considered possible opportunities to construct proofs. Furthermore, according to the policy documents, students should be proficient in creating informal justification and proof arguments to explain why a mathematical result holds (Trinidad and Tobago, 1994, 2005, 2006).

The current CSEC mathematics curriculum also supports the policy makers' advocacy by stating that students should be able to construct mathematical arguments and critique the arguments of others (CXC, 2012). The issue with this requirement is that it entails some measure of providing supporting reasons to justify claims or geometric calculations. The development of informal arguments promotes the explanatory nature of proof in school mathematics. This is important in the context of proof and reasoning because it provides the possible scaffolding needed to motivate students to develop their informal arguments into logical deductive arguments supporting their mathematical claims. By providing these reasons, students can develop metacognitive skills in making connections to Geometry content when solving for an unknown quantity in a given geometric object. As a result, I used this question in Figure 1 in my interview protocol to determine students' opinions about whether a response to a GCNE question would be considered a proof. Furthermore, given the high stakes of this exam in determining students' future engagement with higher-level mathematics, it is important to investigate whether students consider that the informal explanations required by these exercises, are potential opportunities for constructing a proof.

When I asked Sean the question, "Consider the question taken from the January 2014, CSEC Mathematics examination. The question has a pair of parallel lines with a transversal cutting across the two lines at an angle of 24^0. You are required to find the missing angles and provide reasons supporting your answer. In your opinion why or why not would you consider an answer to this question as a proof?" He responded:

Sean: "*Well thinking about it, the question specified that you must provide supporting reasons for your calculations. In this case, I really think they [the examiners] want you to give a clear explanation supporting why your answer came out to be that way. Well to me explaining why is a proof of your claim. I believe by explaining this the examiners will understand your thinking.*"

Sean reflected on the examiner's requirement of providing supporting reasons for calculations. When Sean stated: "I really think they [the examiners] want you to give a clear explanation supporting why your answer came out to be that way" he suggested that the examiners expected students to provide supporting reasons for the steps taken to calculate the unknown values. As Sean stated "to me explaining why is a proof of your claim" he voiced the opinion that providing an explanation of why a claim is valid qualifies as a proof. Sean explained further the necessity for providing a clear explanation. For example, when he stated, "I believe by explaining the examiners will understand your thinking" Sean suggested that the examiners will understand the line of reasoning students use to compute the unknown values. This latter quote also demonstrated that Sean saw the intellectual need of explaining one's thinking when proving. Sean's quote is representative of the opinions of all students I interviewed for this study. All twenty one (21) students expressed that the calculate and explain type of problems in the CSEC exams are indeed opportunities for them to prove a result through explaining of their reasonings about supporting evidence for their calculations.

V. Conclusion

The students' conceptions of the role of appreciation of mathematics demonstrated their understanding of the intellectual necessity of the mathematical knowledge they acquire at school. Harel and Tall (1991) identified the intellectual necessity principle as a standard for pedagogy that involves presenting subject matter in a way that encourages learners to see its intellectual necessity in their mathematical experiences. Therefore students seemed to understand the role of proof based on their teachers' construction of proof arguments. The habitual inclinations for students to write proofs that provide insight and allow all readers to follow the line of reasoning is important for student's future metacognitive development of proof writing skills. The informal calculate and explain opportunities in the CSEC examinations do provide opportunities for students to construct proof arguments. Although these questions do not explicitly state the word "Prove," they provide the necessary scaffolding to develop rationales for reasoning that could eventually go beyond the border of this activity and evolve into formal proof arguments.

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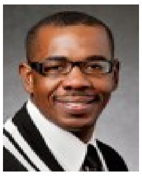
AUTHOR'S PROFILE



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