

Research on the Instructional Design of Cultivating College Students' Mathematics Autonomous Learning Ability Under the Network Environment —— the Teaching of the Concept and Properties of Indefinite Integral as an Example

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Abstract – With the help of the Internet online learning platform, this paper takes the concept and properties of indefinite integral as an example, and proposes the instructional design beneficial to the cultivation of students' autonomous learning ability from three aspects: before class, during class and after class. Step by step to achieve the teaching objectives, highlight the key points, resolve difficulties and enhance students' learning interest, so that students develop good learning habits.

Keywords – Instructional Design, Online Learning Platform, Teaching objectives, Autonomous Learning Ability.

I. INTRODUCTION

Autonomous learning is a learning method in which students decide their learning content, learning method, learning intensity and evaluate their learning effect. It can also be understood as students' ability and habit to guide, control and adjust their learning behavior [1]. College students with good autonomous learning ability, not only can promote the initiative of college students to study, improve learning efficiency, but also beneficial to the enhancement of students' awareness of inquiry and the development of their innovation ability. Therefore, training college students to have good autonomous learning ability is an important aspect of achieving the educational goal of "lifelong learning and active learning".

As an important professional basic course in college education, calculus has always been considered as a difficult course for students due to its high abstractness, strict logic and wide application. The traditional teaching of calculus is mainly taught by teachers, while students learn passively. This kind of teaching model pays attention to the result, despises the process, and emphasizes the knowledge transmission, despises the ability training. This phenomenon makes students' personalized learning needs not be well met and their learning initiative cannot be given full play, which is not conducive to the improvement of students' autonomous learning ability. The teaching model has been unable to adapt to the need of the development of the Times. Therefore, how to train students' autonomous learning ability in the teaching of "calculus" has caught the attention of most educators.

With the development of network technology, the Internet platform has begun to be integrated into the calculus teaching. Microlecture, massive open online course and various online learning platforms complement each other with the classroom teaching, creating a new teaching model of calculus. The purpose is to give full play to students' learning initiative and gradually form the consciousness of innovation on the basis of cultivating students' ability to find problems, propose problems, analyze problems and solve problems, so as to improve students' autonomous learning ability. But how to make full use of these network teaching resources, make it with classroom teaching

organic combination; How to design the teaching process and guide students to think positively, which not only enable the students to master the knowledge they have learned, but also give full play to the students' subjective initiative and constantly improve the students' autonomous learning ability, these have always been a subject for educators to explore. This paper will combine with the online learning platform, take the concept and properties of indefinite integrals as an example, and proposes the instructional design beneficial to the cultivation of students' autonomous learning ability from three aspects: before class, during class and after class.

II. INSTRUCTIONAL DESIGN OF THE CONCEPT AND PROPERTIES OF INDEFINITE INTEGRALS

In order to carry on the instructional design of the concept and properties of indefinite integrals, it is necessary to make clear the teaching objectives, key points and difficult points of this section. Pre-class, in-class and after-class instructional design should revolve around these three aspects. In the process of realizing the teaching objectives step by step, the key point of teaching should be highlighted and the difficult point of teaching should be solved gradually.

The teaching objectives of this lesson are: to understand the concept of the antiderivative, to know the existence of the antiderivative, to clarify the non-uniqueness of the antiderivative ; to understand the concept of the indefinite integral, to clarify the properties of the indefinite integral, to master the basic integral formulas , and can use the basic integral formulas to calculate simple indefinite integral.

Teaching key points are: the concept of the antiderivative and the indefinite integral, the relationship between $\int f(x)dx$ and $f(x)$, the properties of the indefinite integral, use the basic integral formulas to calculate simple indefinite integral.

Teaching difficult points are: understanding and application of formulas $\frac{d}{dx}[\int f(x)dx]=f(x)$, $d[\int f(x)dx]=f(x)dx$ and $\int F'(x)dx = \int dF(x) = F(x) + C$; use the basic integral formulas to calculate simple indefinite integral.

A. Design before Class

Before class, the teacher divides the students into several groups in the first place. The purpose is to let the students learn cooperatively in groups and cultivate their ability of cooperative inquiry.

Then, the teacher asks the students such an inquiry question: suppose that there is a curve passing through the point $(e^2, 3)$, and the slope of the tangent line to the curve at an arbitrary point equals to the reciprocal of the x coordinate of the point of tangency. Find the equation for the curve.

The purpose of designing this problem is to let students discover that it is an inverse problem with respect to derivatives, which lays a foundation for the next step of learning the concept of the indefinite integral, and it is conducive to cultivating students' ability to find problems, put forward problems and mathematical modeling.

The students can write the equations $y' = 1/x$ after exploring. But how to solve for y is a problem for them, so some students realized that it is an inverse operation of derivatives. Some students guess that the equation of the curve is $y = \ln x$, but the curve does not pass through the point $(e^2, 3)$.

Next, the teacher will release the recorded teaching videos about the concept and properties of the indefinite in-

-tegral, the teaching objectives, teaching key points and teaching difficult points to the online learning platform, at the same time, the students will be asked to watch with the following questions:

- (i) What is the antiderivative?
- (ii) Under what conditions does function have an antiderivative? Is the antiderivative unique?
- (iii) If the antiderivative is not unique, how should the antiderivative be represented?
- (iv) What is the relationship between the indefinite integral and the antiderivative?
- (v) What is the relationship between $\int f(x)dx$ and $f(x)$?
- (vi) What are the properties of the indefinite integral?
- (vii) What formulas are included in the basic integral formulas?

The length of teaching video is about 10 minutes, too long time can easily cause students' thinking burnout and discourage their learning enthusiasm. The content of video includes the concept of the antiderivative, the existence and non-uniqueness of the antiderivative; concept of the indefinite integrals, the basic integral formulas, properties of the indefinite integrals and simple calculation problems of the indefinite integrals:

$$\int \sqrt{x}(x^2 - 5) dx ; \int \frac{(x-1)^3}{x^2} dx ; \int (e^x - 3 \cos x) dx ; \int \sec x(\sec x - \tan x) dx .$$

What needs to be pointed out is that the setting of video content should not be too difficult, and students should be able to understand it after a little thought. The purpose is to enhance students' self-confidence, and let students have a sense of the joy of success after watching the teaching video.

The purpose of designing the above seven questions is to guide students to watch the teaching video with clear objectives. These seven questions cover the teaching objectives of this class. If students can answer these seven questions correctly, it shows that they have a preliminary understanding of the main content of this class; it also helps students form the habit of studying with questions.

After watching the teaching video, the students also need to complete the corresponding test questions online to test the effect of online learning and provide the basis for teachers to carry out classroom design in the next step. At the same time, students are required to use their knowledge to complete the inquiry questions mentioned earlier.

For example, in order to test students' mastery of concepts, the following questions are raised in the test questions:

- (i) If $\int f(x)dx = x \cos x + C$, then $f(x) = \underline{\hspace{2cm}}$,
- (ii) $d \left(\int \frac{\cos x}{1 + \sin^2 x} dx \right) = \underline{\hspace{2cm}}$,
- (iii) If x^2 is an antiderivative of a function $f(x)$, then $\int f'(x)dx = \underline{\hspace{2cm}}$.

In order to test students' mastery of computational problems, the following questions are raised in the test questions:

(iv) $\int \frac{(1-x)^2}{\sqrt{x}} dx$; (v) $\int \left(\frac{3}{x^2+1} - \frac{2}{\sqrt{1-x^2}}\right) dx$; (vi) $\int \cos x(\sec x + \tan x) dx$.

B. Class Design

The objectives of classroom teaching are to further to deepen students' understanding of the concepts of the antiderivative and the indefinite integrals on the basis of pre-class learning, and further learn to calculate indefinite integrals by using the basic integral formulas.

In class, teachers first select a study group and send representatives to report the learning results of watching teaching video before class, that is, to answer the questions (i) ~ (vii), test questions (i) ~ (vi) and inquiry questions raised before class. This is conducive to cultivating students' ability to express and communicate in mathematical language. If students do not explain the correct, the teacher can organize the class to discuss, and then the teacher to give comments; if all the explanations are correct, the teacher can also ask the group to answer questions that arise when the other groups members watched the instructional video, and then the teacher gives a comment.

Next, in order to deepen students' understanding of the concept, the teacher gives the following exercises in the basis of test questions (i)~(iii) :

(i) If e^{-x} is an antiderivative of a function $f(x)$, then $\int x^2 f(\ln x) dx =$ _____ ;

(ii) If a function $f(x)$ is an antiderivative of e^{-x} , then $\int \frac{f(\ln x)}{x} dx =$ _____ ;

(iii) If $\int df(x) = \int dg(x)$, Then () is not true in the following equation.

(A) $f(x)=g(x)$; (B) $f'(x)=g'(x)$; (C) $df(x)=dg(x)$; (D) $d(\int f'(x)dx) = d(\int g'(x)dx)$.

(iv) Let $\int \frac{x^2}{\sqrt{1-x^2}} dx = ax\sqrt{1-x^2} + b\int \frac{dx}{\sqrt{1-x^2}}$ and a, b are constants, find a, b .

The first two questions examined the concept of the antiderivative, both involving functions $f(x)$ and e^{-x} , and both the indefinite integrals contain $f(\ln x)$. Students are mainly confused whether the derivative of $f(x)$ is e^{-x} or the derivative of e^{-x} is $f(x)$. The third question is a choice question about the relationship between antiderivatives. It mainly examines the properties of the indefinite integrals, and this question is a bit abstract. The teacher can guide students to combine the properties of the indefinite integral and the method of specialization for thinking. The fourth question is relatively difficult. The teachers can inspire students to make use of observation, comparison, association and other methods to establish connection with the previously learned content and find solutions.

For these four questions, the teacher first gives the students 15 minutes to think independently. If the students fail to think independently under the proper prompts of the teacher, or only a few students come to the correct conclusion, then let the students discuss and explore in groups, and then choose the student representatives to report. After discussion, the conclusion will leave a deep impression on students. At the same time, the process of discussion can also stimulate students' learning initiative, which is much better than the teacher's simple explanation. For the fourth question, since the solution method is not unique, the teacher can guide students to discuss from different angles and carry out multiple ways to solve a problem.

Since the calculation of indefinite integrals is the key content, the examples of using the basic integral formulas to calculate indefinite integrals in the teaching video are relatively simple and lack of skill. Therefore, in order to further learn the method of integration by substitution later, and to cultivate the flexibility of students' thinking, students need to be trained in the skills of computing the indefinite integral. Therefore, teachers can give the following calculation questions on the basis of test questions (iv)~(vi):

$$(i) \int \frac{2x^4 + x^2 + 3}{x^2 + 1} dx; (ii) \int \sin^2 \frac{x}{2} dx; (iii) \int \frac{1}{\sin^2 x \cdot \cos^2 x} dx; (iv) \int \tan^2 x dx.$$

These four problems cannot be calculated directly by using the basic integral formulas. Teachers can ask students to work in groups and discuss how to change them through identity deformation into a form that can be calculated using the basic integral formulas. Students may be unable to find a breakthrough point at the beginning. The teacher can prompt students. For example, in the calculation questions (i), the integrand is a rational improper fraction, which can be decomposed into a polynomial plus and a proper fraction; in the calculation questions (ii)~(iv), the integrand is related to trigonometric functions, we can choose an appropriate trigonometric formula to convert it into a form containing a basic integral formula. After the teacher prompts and the discussion between the students, the students can come to the correct conclusion. Then the teacher chooses a student representative to write the correct conclusion on the blackboard. After that, the teacher can also give some exercises for students to consolidate what they have learned.

At last, the teacher can introduce the concept of a differential equation, combine the relevant content of the differential equation to expand students' knowledge, give examples related to real life, and let students feel the application of indefinite integral in practical problems.

Example An object starts from static and moves in a straight line, its speed is $3t^2$ (Unit: m/s) at time t (Unit: s).

- (i) What is the distance from the starting point at time $t = 3$?
- (ii) How long will it take the object to finish the distance of 360 m?

In the process of solving this problem, students have gone through the steps of putting forward hypotheses, setting up differential equations, solving differential equations and so on. In the process of experiencing the perfect integration of indefinite integral and solving differential equations, students have also gone through the process of simple mathematical modeling, which is beneficial to the cultivation of mathematical application consciousness and innovation consciousness of students.

C. After class design

After class, the teacher asks the students to summarize the knowledge and methods learned in this lesson, and upload the summary report to the group of classes in the online learning platform for sharing. The form of summary can be varied, such as a written report, Power Point, and mind map. The teacher will grade the students' summary report, which will be included in the usual performance. The purpose of this design is to enable students to develop good habits of reflective learning, to consolidate knowledge, strengthen memory and improve autonomous learning ability. At the same time, teachers should let students complete the corresponding test questions on the online learning platform to test students' classroom learning effect and provide a basis for teaching reflection. For individual problems reflected in the test, teachers can use the online learning platform to guide

students individually. For the common problems reflected in the test, teachers can discuss them online with all their classmates, or can stay in the next class to explain them.

III. CONCLUSION

This teaching design based on students, pays attention to the development of students' thinking process, with the help of online learning platform, step-by-step inspires and guides students to think positively, fully promoting the students' subjective initiative, generates knowledge independently in the process of independent exploration and cooperative learning, and achieves teaching objectives. The teaching design also pays attention to the cultivation of the students' application consciousness and innovation consciousness, pays attention to the extension of the knowledge, enhances the students' interest in learning and gradually improves students' independent learning ability while fostering good learning habits.

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